

**Tabla A-1.** Parejas de la transformada de Laplace.

	$f(t)$	$F(s)$
1	Unidad de impulso $\delta(t)$	1
2	Unidad de paso $1(t)$	$\frac{1}{s}$
3	$t$	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
6	$e^{-at}$	$\frac{1}{s+a}$
7	$te^{-at}$	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\text{sen } \omega t$	$\frac{\omega}{s^2 + \omega^2}$
11	$\text{cos } \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$\text{sinh } \omega t$	$\frac{\omega}{s^2 - \omega^2}$
13	$\text{cosh } \omega t$	$\frac{s}{s^2 - \omega^2}$
14	$\frac{1}{a} (1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a} (e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a} (be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab} \left[ 1 + \frac{1}{a-b} (be^{-at} - ae^{-bt}) \right]$	$\frac{1}{s(s+a)(s+b)}$

(continúa)

Tabla A-1. (Continuación).

	$f(t)$	$F(s)$
18	$\frac{1}{a^2} (1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2} (at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at} \operatorname{sen} \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
21	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \operatorname{sen} \omega_n \sqrt{1-\zeta^2} t \quad (0 < \zeta < 1)$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \operatorname{sen} (\omega_n \sqrt{1-\zeta^2} t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, 0 < \phi < \pi/2)$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
24	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \operatorname{sen} (\omega_n \sqrt{1-\zeta^2} t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, 0 < \phi < \pi/2)$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
25	$1 - \cos \omega t$	$\frac{\omega^2}{s(s^2 + \omega^2)}$
26	$\omega t - \operatorname{sen} \omega t$	$\frac{\omega^3}{s^2(s^2 + \omega^2)}$
27	$\operatorname{sen} \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2 + \omega^2)^2}$
28	$\frac{1}{2\omega} t \operatorname{sen} \omega t$	$\frac{s}{(s^2 + \omega^2)^2}$
29	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
30	$\frac{1}{\omega_2^2 - \omega_1^2} (\cos \omega_1 t - \cos \omega_2 t) \quad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$
31	$\frac{1}{2\omega} (\operatorname{sen} \omega t + \omega t \cos \omega t)$	$\frac{s^2}{(s^2 + \omega^2)^2}$

**Tabla A-2.** Propiedades de la transformada de Laplace.

1	$\mathcal{L}[Af(t)] = AF(s)$
2	$\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathcal{L}_{\pm} \left[ \frac{d}{dt} f(t) \right] = sF(s) - f(0_{\pm})$
4	$\mathcal{L}_{\pm} \left[ \frac{d^2}{dt^2} f(t) \right] = s^2F(s) - sf(0_{\pm}) - \dot{f}(0_{\pm})$
5	$\mathcal{L}_{\pm} \left[ \frac{d^n}{dt^n} f(t) \right] = s^nF(s) - \sum_{k=1}^n s^{n-k} f^{(k-1)}(0_{\pm})$ donde $f^{(k-1)} = \frac{d^{k-1}}{dt^{k-1}} f(t)$
6	$\mathcal{L}_{\pm} \left[ \int f(t) dt \right] = \frac{F(s)}{s} + \frac{1}{s} \left[ \int f(t) dt \right]_{t=0_{\pm}}$
7	$\mathcal{L}_{\pm} \left[ \int \dots \int f(t) (dt)^n \right] = \frac{F(s)}{s^n} + \sum_{k=1}^n \frac{1}{s^{n-k+1}} \left[ \int \dots \int f(t) (dt)^k \right]_{t=0_{\pm}}$
8	$\mathcal{L} \left[ \int_0^t f(t) dt \right] = \frac{F(s)}{s}$
9	$\int_0^{\infty} f(t) dt = \lim_{s \rightarrow 0} F(s)$ si $\int_0^{\infty} f(t) dt$ salidas
10	$\mathcal{L}[e^{-at} f(t)] = F(s + a)$
11	$\mathcal{L}[f(t - \alpha)1(t - \alpha)] = e^{-\alpha s} F(s) \quad a \geq 0$
12	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$
13	$\mathcal{L}[t^2 f(t)] = \frac{d^2}{ds^2} F(s)$
14	$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s) \quad (n = 1, 2, 3, \dots)$
15	$\mathcal{L} \left[ \frac{1}{t} f(t) \right] = \int_s^{\infty} F(s) ds$ si $\lim_{t \rightarrow 0} \frac{1}{t} f(t)$ salidas
16	$\mathcal{L} \left[ f \left( \frac{1}{a} \right) \right] = aF(as)$
17	$\mathcal{L} \left[ \int_0^t f_1(t - \tau) f_2(\tau) d\tau \right] = F_1(s)F_2(s)$
18	$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_{c-j\omega}^{c+j\omega} F(p)G(s - p) dp$

Por último se presentan dos teoremas frecuentemente utilizados junto con las transformadas de Laplace de la función pulso y de la función impulso.

Teorema de valor inicial	$f(0+) = \lim_{t \rightarrow 0+} f(t) = \lim_{s \rightarrow \infty} sF(s)$
Teorema de valor final	$f(\infty) = \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$
<p>Función pulso</p> $f(t) = \frac{A}{t_0} 1(t) - \frac{A}{t_0} 1(t - t_0)$	$\mathcal{L}[f(t)] = \frac{A}{t_0 s} - \frac{A}{t_0 s} e^{-st_0}$
<p>Función impulso</p> $g(t) = \lim_{t_0 \rightarrow 0} \frac{A}{t_0}, \quad \text{para } 0 < t < t_0$ $= 0, \quad \text{para } t < 0, t_0 < t$	$\begin{aligned} \mathcal{L}[g(t)] &= \lim_{t_0 \rightarrow 0} \left[ \frac{A}{t_0 s} (1 - e^{-st_0}) \right] \\ &= \lim_{t_0 \rightarrow 0} \frac{\frac{d}{dt_0} [A(1 - e^{-st_0})]}{\frac{d}{dt_0} (t_0 s)} \\ &= \frac{As}{s} = A \end{aligned}$