AADECA

Asociación Argentina
de Control Automático

/ /

→ Solución parcial nº1: MPIE (30/04/2025)

Ej. 1:

$$\frac{|z+1|}{|z-2|}=4 \Rightarrow |z+1| = 4|z-2| \quad ; \text{ si } z = x+iy$$

$$|x+iy+1| = 4|x+iy-2| \Rightarrow |(x+1)+iy| = 4|(x-2)+iy|$$

por def. de módulo $\Rightarrow \sqrt{(x+1)^2+y^2} = 4\sqrt{(x-2)^2+y^2} \Rightarrow$

$$(x+1)^2+y^2 = 16\cdot\left((x-2)^2+y^2\right)$$

$$x^2+2x+1+y^2 = 16\cdot(x^2-4x+4+y^2) = 16x^2-64x+64+16y^2$$

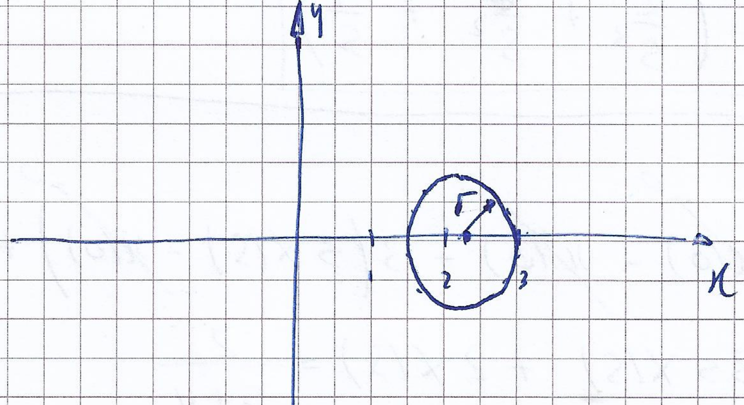
$$\Rightarrow 15x^2-66x+63+15y^2=0$$

$$x^2-\frac{22}{5}x+\frac{21}{5}+y^2=0 \qquad \left|\left(x-11/5\right)^2 = x^2-\frac{22}{5}x+\frac{121}{25}\right.$$

$$\Rightarrow (x-11/5)^2-\frac{121}{25}+\frac{21}{5}+y^2=0$$

$$(x-11/5)^2-\frac{16}{25}+y^2 \Rightarrow (x-11/5)^2+y^2 = \frac{16}{25} = r^2$$

$$\Rightarrow \text{círculo} \Rightarrow \boxed{(x-11/5)^2+y^2=\left(\frac{4}{5}\right)^2}$$



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Ejercicio 2 $\quad w = \alpha z + \beta$

$$z = 1+i \longrightarrow w = i$$
$$z = -1 \longrightarrow w = 1+i$$

a). $\left.\begin{array}{l} i = \alpha\cdot(1+i) + \beta \\ 1+i = \alpha\cdot(-1) + \beta \end{array}\right\} \Rightarrow 1+i = -\alpha + \beta \Rightarrow \beta = 1+i+\alpha$

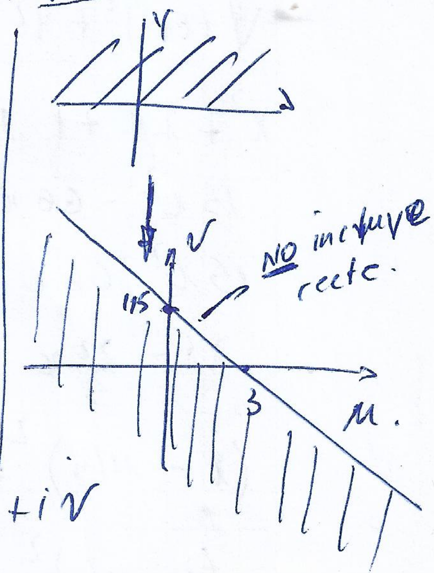
Reemp $\Rightarrow \quad i = \alpha\cdot(1+i) + 1+i+\alpha = \alpha\cdot(1+i+1) + 1+i$

$$\alpha\cdot(2+i) = -1 \Rightarrow \alpha = \frac{-1}{2+i} = \frac{1\,\angle 180^\circ}{2{,}236\,\angle 26{,}565} = 0{,}447\,\angle 153{,}435$$

$$\boxed{\alpha = -\frac{2}{5} + \frac{1}{5}i}$$

$$\beta = 1+i-\frac{2}{5}+\frac{1}{5}i \Rightarrow \boxed{\beta = \frac{3}{5} + \frac{6}{5}i}$$

b). $w = \left(-\frac{2}{5}+\frac{1}{5}\cdot i\right)\cdot z + \left(\frac{3}{5}+\frac{6}{5}\cdot i\right) = u + iv$



$z = x+iy \Rightarrow$

$$w = u+iv = \left(-\frac{2}{5}+\frac{1}{5}i\right)\cdot(x+iy) + \left(\frac{3}{5}+\frac{6}{5}\cdot i\right)$$
$$= -\frac{2}{5}\cdot x - i\frac{2}{5}\cdot y + i\frac{1}{5}\cdot x + i^2\cdot\frac{1}{5}\cdot y + \frac{3}{5} + i\frac{6}{5}$$
$$= -\frac{2}{5}\cdot x - i\frac{2}{5}\cdot y + i\frac{1}{5}x - \frac{1}{5}\cdot y + \frac{3}{5} + i\frac{6}{5}$$
$$= \left(-\frac{2}{5}\cdot x - \frac{1}{5}\cdot y + \frac{3}{5}\right) + i\left(\frac{1}{5}\cdot x - \frac{2}{5}\cdot y + \frac{6}{5}\right) = u+iv$$

$$(-2x - y + 3) + i(x - 2y + 6) = 5\cdot(u+iv) = 5\cdot u + i5v$$

$$\Rightarrow \left\{\begin{array}{l} \boxed{5u = -2x - y + 3} \Rightarrow y = -2x + 3 - 5u \\ \boxed{5v = x - 2y + 6} \Rightarrow x = 5v + 2y - 6 \end{array}\right.$$

$$\Rightarrow y = -2\cdot(5v + 2y - 6) + 3 - 5u = -10v - 4y + \overbrace{12 + 3}^{15} - 5u$$

$$5y = -10v - 5u + 15 \Rightarrow \boxed{y = -2v - u + 3} \Longrightarrow y > 0$$

$$\Rightarrow \operatorname{Im}(z) = y > 0 \Rightarrow -2v - u + 3 > 0 \Rightarrow \boxed{-2v - u > -3}$$

c) ptos fijos $\Rightarrow$ $w = z$

$$w = z = \left(-\frac{2}{5} + \frac{1}{5}.i\right) z + \frac{3}{5} + \frac{6}{5} i$$

$$z - \left(-\frac{2}{5} + \frac{1}{5}.i\right).z = \frac{3}{5} + \frac{6}{5}.i$$

$$z.\left(1 + \frac{2}{5} - \frac{1}{5}.i\right) = \frac{3}{5} + \frac{6}{5}.i = z\left(\frac{7}{5} - i\frac{1}{5}\right)$$

$$\Rightarrow \boxed{z = \frac{3}{10} + \frac{9}{10}.i}$$

Ejercicio 3: $f(t) = 2t^2 H(t-3)$

2do teorema del Despl.

$$\mathcal{L}(f(t-a).H(t-a)) = e^{-a.s} F(s)$$

$$F(s) = \mathcal{L}(f(t))$$

$$f(t) = 2.(\underbrace{t-3}_{a} + \underbrace{3}_{b})^2 H(t-3)$$

$$f(t) = 2.((t-3)^2 + 6.(t-3) + 9)\, H(t-3)$$

$$f(t) = 2.(t-3)^2.H(t-3) + 12.(t-3).H(t-3) + 18\, H(t-3)$$

$$\mathcal{L}(f(t)) = F(s) = 2.\mathcal{L}((t-3)^2.H(t-3)) + 12\,\mathcal{L}(t-3).H(t-3) + 18\,\mathcal{L}\,H(t-3)$$

$$F(s) = 2e^{-3s}\frac{2!}{s^3} + 12\,e^{-3s}\frac{1}{s^2} + 18.e^{-3s}.\frac{1}{s}$$

$$\boxed{F(s) = 2e^{-3s}\left(\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}\right)}$$

Ejercicio 4:

a)- $s^2X(s) - s\cancel{x(0)}^{0} - \cancel{x'(0)}^{1} - 3(sX(s) - \cancel{x(0)}^{0}) + 2X(s) = 2\frac{1}{s+4}$

$$s^2X(s) - 1 - 3sX(s) + 2X(s) = \frac{2}{s+4}$$

$$X(s).\underbrace{(s^2 - 3s + 2)} = \frac{2}{s+4} + 1 = \frac{2+s+4}{s+4} = \frac{s+6}{s+4}$$

$$s_{2,3} = \frac{3 \pm \sqrt{9-4.1.2}}{2.1} \quad s_2 = 2,\; s_3 = 1 \Rightarrow X(s).(s-2).(s-1) = \frac{s+6}{s+4}$$

$$X(s) = \frac{s+6}{(s+4)\cdot(s-2)\cdot(s-1)} = \frac{a}{s+4} + \frac{b}{s-2} + \frac{c}{s-1}$$

$$\Rightarrow p/\ s=-4 \Rightarrow \left.\frac{s+6}{(s-2)\cdot(s-1)}\right|_{s=-4} = a = \frac{2}{(-6)\cdot(-5)} \Rightarrow \boxed{a = \frac{1}{15}}$$

$$p/\ s=2 \Rightarrow \left.\frac{s+6}{(s+4)\cdot(s-1)}\right|_{s=2} = b = \frac{8}{6\cdot 1} \Rightarrow \boxed{b = \frac{4}{3}}$$

$$p/\ s=1 \Rightarrow \left.\frac{s+6}{(s+4)\cdot(s-2)}\right|_{s=1} = c = \frac{7}{5\cdot(-1)} \Rightarrow \boxed{c = -\frac{7}{5}}$$

$$\Rightarrow x(t) = \mathcal{L}^{-1}\left(X(s)\right) = \frac{1}{15}\mathcal{L}^{-1}\left(\frac{1}{s+4}\right) + \frac{4}{3}\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) - \frac{7}{5}\mathcal{L}^{-1}\left(\frac{1}{s-1}\right)$$

$$\boxed{x(t) = \frac{1}{15}\cdot e^{-4t} + \frac{4}{3}e^{2t} - \frac{7}{5}e^{t}}$$

b) i) $s^2 X(s) - s\,x(0) - x'(0) - 3\left(sX(s) - x(0)\right) + 2X(s) = U(s)$

$$\underbrace{x(0) = 0,\quad x'(0) = 0,\quad x(0) = 0}_{p/\ \text{función de transf.: condiciones iniciales } = 0}$$

$$X(s)\cdot(s^2 - 3s + 2) = U(s)$$

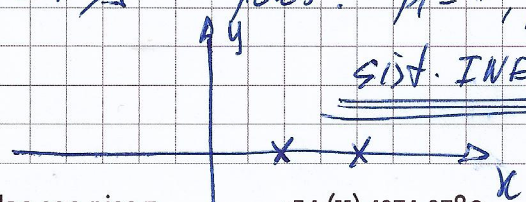
$$\Rightarrow \boxed{G(s) = \frac{X(s)}{U(s)} = \frac{1}{s^2-3s+2}}$$

ii). Ec. característica:

$$\boxed{s^2 - 3s + 2 = 0}$$

iii) ceros $= \nexists$ polos: $p_1 = 1,\ p_2 = 2$

Sist. INESTABLE



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