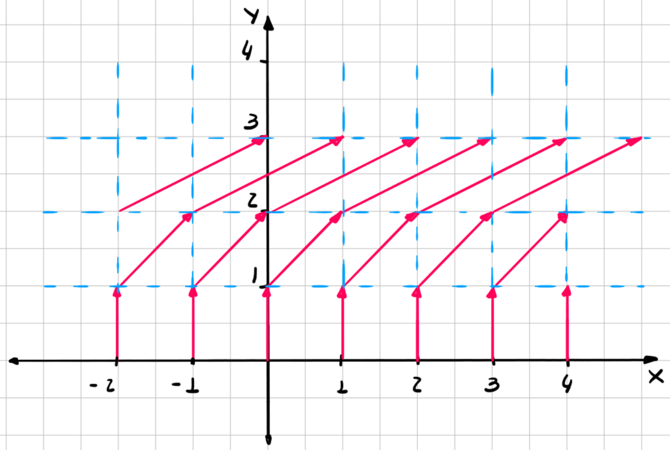
TP N° 3: "Funciones con valores vectoriales - Campos Vectoriales"

PROBLEMA 1:

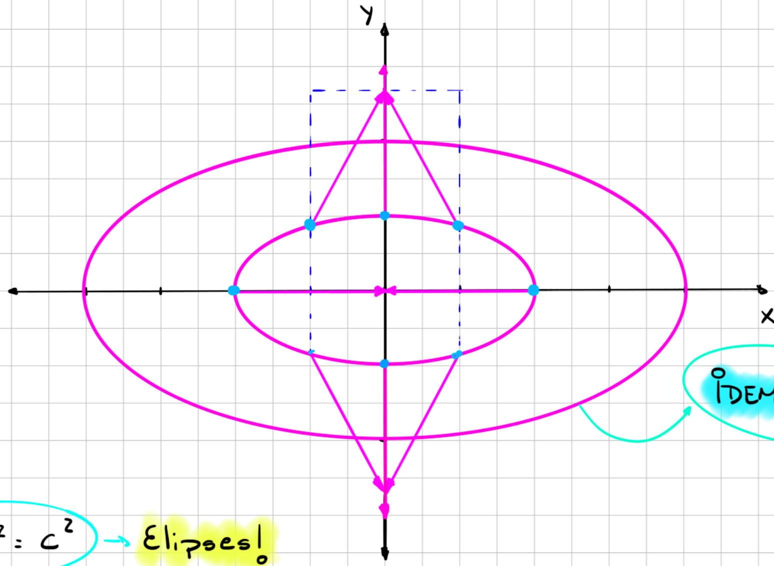
a)- $\bar{F}(x,y) = y\,\bar{i} + \bar{j}$

$P(x,y)$	$\bar{F}(x,y)$
$(0;0)$	$(0\bar{i} + \bar{j})$
$(1;0)$	$(0\bar{i} + \bar{j})$
$(-1;0)$	$(0\bar{i} + \bar{j})$
$(2;0)$	$(0\bar{i} + \bar{j})$
$(0;1)$	$(\bar{i} + \bar{j})$
$(1;1)$	$(\bar{i} + \bar{j})$
$(0;2)$	$(2\bar{i} + \bar{j})$
$(1;2)$	$(2\bar{i} + \bar{j})$



b) $\bar{F}(x,y) = -x\,\bar{i} + 2y\,\bar{j}$

$P(x,y)$	$\bar{F}(x,y)$
$(0;0)$	$(0\bar{i} + 0\bar{j})$
$(0;1)$	$(0\bar{i} + 2\bar{j})$
$(-2;0)$	$(2\bar{i} + 0\bar{j})$
$(2;0)$	$(-2\bar{i} + 0\bar{j})$
$(0;-1)$	$(0\bar{i} - 2\bar{j})$
$(1;0{,}87)$	$(-\bar{i} + 1{,}74\bar{j})$
$(1;-0{,}87)$	$(-\bar{i} - 1{,}74\bar{j})$
$(4;0)$	$(-4\bar{i} + 0\bar{j})$
$(-4;0)$	$(4\bar{i} + 0\bar{j})$
$(0;2)$	$(0\bar{i} + 4\bar{j})$
$(0;-2)$	$(0\bar{i} - 4\bar{j})$



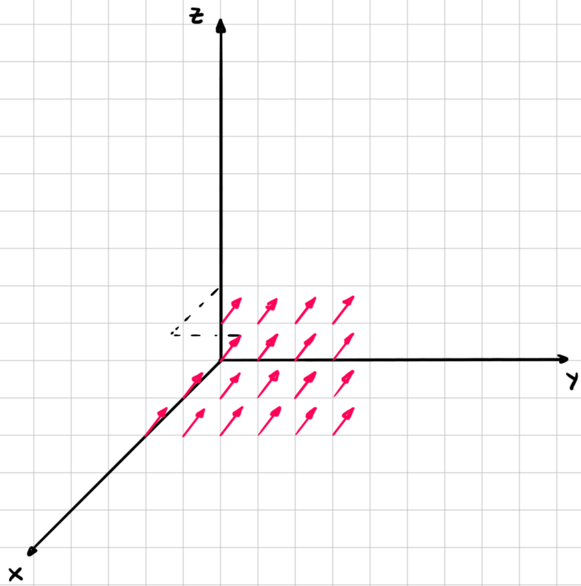
ÍDEM $C = 4$

$|\bar{F}| = \sqrt{x^2 + 4y^2} = c \Rightarrow x^2 + 4y^2 = c^2$ → ¡Elipses!

$\Rightarrow x^2 + 4y^2 = c^2 \Rightarrow c = 2 \Rightarrow x^2 + 4y^2 = 4 \Rightarrow \frac{x^2}{4} + y^2 = 1 \quad \Rightarrow c = 4 \Rightarrow x^2 + 4y^2 = 16 \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$

c)- $\bar{F}(x,y,z) = \bar{i} + \bar{j} + \bar{k}$

$P(x,y,z)$	$\bar{F}(x,y,z)$
$(0;0;0)$	$\bar{i} + \bar{j} + \bar{k}$
$(0;0;1)$	$\bar{i} + \bar{j} + \bar{k}$
$(0;0;-1)$	$\bar{i} + \bar{j} + \bar{k}$
$(0;1;0)$	$\bar{i} + \bar{j} + \bar{k}$
$(0;-1;0)$	$\bar{i} + \bar{j} + \bar{k}$
$(0;2;0)$	$\bar{i} + \bar{j} + \bar{k}$



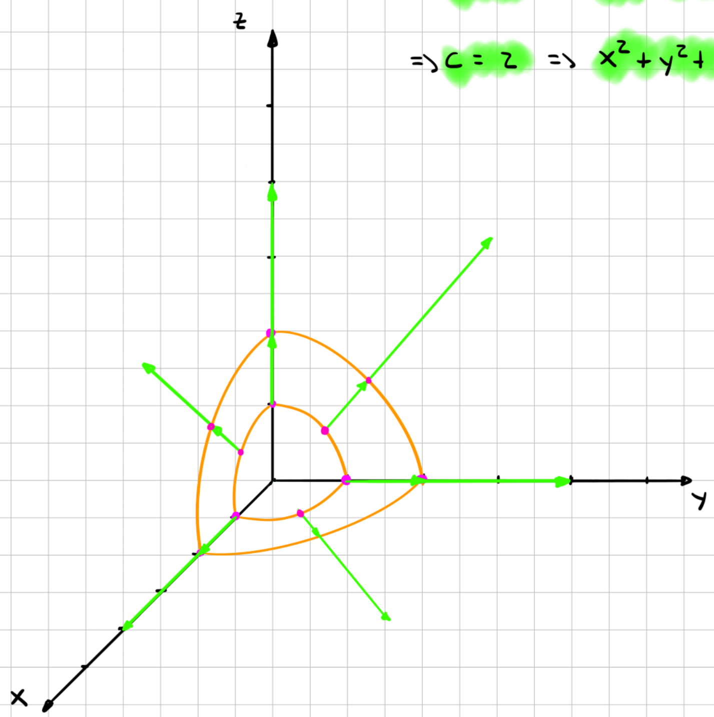
d).- $\bar{F}(x,y,z) = x\bar{i} + y\bar{j} + z\bar{k}$

$P(x,y,z)$	$\bar{F}(x,y,z)$
$(0,1,0)$	$0\bar{i} + \bar{j} + 0\bar{k}$
$(1,0,0)$	$\bar{i} + 0\bar{j} + 0\bar{k}$
$(0,0,1)$	$0\bar{i} + 0\bar{j} + \bar{k}$
$(2,0,0)$	$2\bar{i} + 0\bar{j} + 0\bar{k}$
$(0,2,0)$	$0\bar{i} + 2\bar{j} + 0\bar{k}$
$(0,0,2)$	$0\bar{i} + 0\bar{j} + 2\bar{k}$

$|\bar{F}| = \sqrt{x^2+y^2+z^2} = C \Rightarrow x^2+y^2+z^2 = C$

$\Rightarrow C = 1 \Rightarrow x^2+y^2+z^2 = 1$

$\Rightarrow C = 2 \Rightarrow x^2+y^2+z^2 = 4$



PROBLEMA 2:

a).- $f(x,y) = e^{3x}\cos(4y)$

$$\bar{\nabla} f = \frac{\partial f}{\partial x}\bar{i} + \frac{\partial f}{\partial y}\bar{j} = 3e^{3x}\cos(4y)\,\bar{i} - 4e^{3x}\operatorname{Sen}(4y)\,\bar{j}$$

b).- $f(x,y) = 5x^2 + 3xy + 10y^2$

$$\nabla f = (10x + 3y)\bar{i} + (3x + 20y)\bar{j}$$

c).- $f(x,y,z) = \frac{y}{z} + \frac{z}{x} - \frac{xz}{y}$

$$\bar{\nabla} f = \frac{\partial F}{\partial x}\bar{i} + \frac{\partial F}{\partial y}\bar{j} + \frac{\partial F}{\partial z}\bar{k} = \left(-\frac{z}{x^2} - \frac{z}{y}\right)\bar{i} + \left(\frac{1}{z} + \frac{xz}{y^2}\right)\bar{j} + \left(\frac{-y}{z^2} + \frac{1}{x} - \frac{x}{y}\right)\bar{k}$$

d).- $f(x,y,z) = xy \cdot \ln(xy) + z$

$$\bar{\nabla} f = \left[y \cdot \ln(xy) + \cancel{x}y\,\frac{y}{\cancel{xy}}\right]\bar{i} + \left[x\ln(xy) + x\cancel{y}\,\frac{x}{\cancel{xy}}\right]\bar{j} + \bar{k}$$

$$\bar{\nabla} f = \left[y\ln(xy) + y\right]\bar{i} + \left[x\ln(xy) + x\right]\bar{j} + \bar{k}$$

PROBLEMA (3):

Determinar: 1)_ ROTACIONAL

2)_ DIVERGENCIA

a)_ $\bar{F}(x,y,z) = (xyz)\bar{i} + y\bar{j} + z\bar{k}$

$$\text{ROTOR}: \bar{\nabla}\times\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\bar{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\bar{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\bar{k}$$

$$\text{ROTOR} = (0-0)\bar{i} - (0-xy)\bar{j} + (0-xz)\bar{k}$$

$$\text{ROTOR}: 0\bar{i} + xy\,\bar{j} - xz\,\bar{k}$$

$$\text{DIVERGENCIA} = \bar{\nabla}\cdot\bar{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} = yz + 1 + 1 = 2 + yz$$

b)_ $\bar{F}(x,y,z) = \operatorname{Sen}(x)\bar{i} + \operatorname{Cos}(x)\bar{j} + z^2 k$

$$\text{ROTOR} = (0-0)\bar{i} - (0-0)\bar{j} + (-\operatorname{Sen}(x) - 0)\bar{k}$$

$$\text{ROTOR}: 0\bar{i} + 0\bar{j} - \operatorname{Sen}(x)\bar{k}$$

$$\operatorname{div}(\bar{F}) = \operatorname{Cos}(x) + 2z$$

c)_ $\bar{F}(x,y,z) = e^{-xyz}(\bar{i} + \bar{j} + \bar{k})$

$$\text{ROTOR} = (-xz\,e^{-xyz} + xy\,e^{-xyz})\bar{i} - (-yz\,e^{-xyz} + xy\,e^{-xyz})\bar{j} + (-yz\,e^{-xyz} + xz\,e^{-xyz})\bar{k}$$

$$\text{ROTOR}: \left[xe^{-xyz}(y-z)\right]\bar{i} + \left[ye^{-xyz}(z-x)\right]\bar{j} + \left[z e^{-xyz}(x-y)\right]\bar{k}$$

$$\operatorname{div}(\bar{F}) = -yz\,e^{-xyz} - xz\,e^{-xyz} - xy\,e^{-xyz}$$

$$\operatorname{div}(F) = -e^{-xyz}(yz + xz + xy)$$

PROBLEMA (4):

a)_ $\bar{F}(x,y) = (3x^2y)\bar{i} + (x^3)\bar{j}$

$$\left.\begin{array}{l} M = 3x^2y \\ N = x^3 \end{array}\right\} \quad \begin{array}{l} \dfrac{\partial M}{\partial y} = 3x^2 \\ \dfrac{\partial N}{\partial x} = 3x^2 \end{array}$$

Como:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\Rightarrow$ Admite Función POTENCIAL_

Enfoque Conceptual:

$\bar{F}(x,y) = M(x,y)\bar{i} + N(x,y)\bar{j}$

Sí: $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x} \Rightarrow$ Admite Función Potencial..

$\varphi(x,y)$ Es función potencial $\Leftrightarrow \bar{\nabla}\varphi = \bar{F}$

$$\Rightarrow \frac{\partial \varphi}{\partial x}\bar{i} + \frac{\partial \varphi}{\partial y}\bar{j} = M(x,y)\bar{i} + N(x,y)\bar{j}$$

Para el cálculo de $\varphi(x,y)$, planteamos:

$$\varphi(x,y) = \int M\,dx = \int 3x^2y\,dx = \cancel{3}\frac{x^3}{\cancel{3}}y + C(y) = x^3y + C(y) \quad (1)$$

$$\varphi(x,y) = \int N\,dy = \int x^3\,dy = x^3y + h(x) \quad (2)$$

$$(1) = (2) \Rightarrow \cancel{x^3y} + C(y) = \cancel{x^3y} + h(x)$$

$$\Rightarrow C(y) = h(x) = c$$

$$\Rightarrow \varphi(x,y) = x^3y + c$$

b). $\bar{F}(x,y) = \left[y\cos(xy)\right]\bar{i} + \left[x\cos(xy)\right]\bar{j}$

$$\left.\begin{array}{l} M = y\cos(xy) \\ N = x\cos(xy) \end{array}\right\} \quad \left.\begin{array}{l} \frac{\partial M}{\partial y} = \cos(xy) - y\,\mathrm{Sen}(xy)x = \cos(xy) - xy\,\mathrm{Sen}(xy) \\ \frac{\partial N}{\partial x} = \cos(xy) - x\,\mathrm{Sen}(xy)y = \cos(xy) - xy\,\mathrm{Sen}(xy) \end{array}\right\} = \Rightarrow \text{Admite Función Potencial}$$

$$\varphi(x,y) = \int M\,dx = \int \left[y\cos(xy)\right]dx = \int \cos(t)\,dt = \mathrm{Sen}(t) + C(y) = \mathrm{Sen}(xy) + C(y) \quad (1)$$

$$xy = t \Rightarrow y\,dx = dt$$

$$\varphi(x,y) = \int N\,dy = \int \left[x\cos(xy)\right]dy = \int \cos(t)\,dt = \mathrm{Sen}(t) + h(y) = \mathrm{Sen}(xy) + h(y) \quad (2)$$

$$xy = t \Rightarrow x\,dy = dt$$

$$\left.\begin{array}{l} (1) = (2) \Rightarrow \mathrm{Sen}(xy) + C(y) = \mathrm{Sen}(xy) + h(y) \\ \Rightarrow C(y) = h(y) = C \end{array}\right\} \varphi(x,y) = \mathrm{Sen}(xy) + C$$

c). $\bar{F}(x,y,z) = \underset{(M)}{y\bar{i}} + \underset{(N)}{x\bar{j}} + \underset{(P)}{\bar{k}}$

Campo conservativo $\Rightarrow$ ROTOR = 0

$$\text{ROTOR} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\bar{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\bar{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\bar{k}$$

$$\text{ROTOR} = (0-0)\bar{i} - (0-0)\bar{j} + (1-1)\bar{k} = 0 \Rightarrow \text{CONSERVATIVO..}$$

$$\varphi(x,y,z) = \int M\,dx = \int y\,dx = xy + C(y,z) \quad (1)$$

$$\varphi(x,y,z) = \int N\,dy = \int x\,dy = xy + h(x,z) \quad (2)$$

$$\varphi(x,y,z) = \int P\,dz = \int dz = z + g(x,y) \quad (3)$$

$1 = 2 = 3$

$$xy + C(y,z) = xy + h(x,z) = z + g(x,y)$$

$$\Rightarrow \begin{cases} C(y,z) = h(x,z) = z + c \\ g(x,y) = xy + c \end{cases} \Rightarrow \varphi(x,y,z) = xy + z + c$$

d).- $\bar{F}(x,y,z) = x\bar{i} + e^y Sen(z)\,\bar{j} + e^y Cos(z)\,\bar{k}$

$$\text{Rotor} = \left[e^y Cos(z) - e^y Cos(z)\right]\bar{i} - (0-0)\bar{j} + (0-0)\bar{k} = 0 \Rightarrow \text{CONSERVATIVO.-}$$

$$\varphi(x,y,z) = \int M\,dx = \int x\,dx = \frac{x^2}{2} + C(y,z) \quad (1)$$

$$\varphi(x,y,z) = \int N\,dy = \int e^y Sen(z)\,dy = e^y Sen(z) + h(x,z) \quad (2)$$

$$\varphi(x,y,z) = \int P\,dz = \int e^y Cos(z)\,dz = e^y sen(z) + g(x,y) \quad (3)$$

$$1 = 2 = 3 \Rightarrow \frac{x^2}{2} + C(y,z) = e^y Sen(z) + h(x,z) = e^y Sen(z) + g(x,y)$$

$$\left.\begin{array}{l} \Rightarrow C(y,z) = e^y Sen(z) + C \\ \Rightarrow h(x,y) = g(x,y) = \dfrac{x^2}{2} + C \end{array}\right\} \varphi(x,y,z) = \frac{x^2}{2} + e^y Sen\, z + C$$

PROBLEMA 5: Inciso 1

DATOS: $\bar{F}(x,y,z) = \bar{i} + 2x\bar{j} + 3y\bar{k}$, $\bar{G}(x,y,z) = x\bar{i} - y\bar{j} + z\bar{k}$

a).- $Rot(\bar{F}\times\bar{G})$

$$\bar{F}\times\bar{G} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2x & 3y \\ x & -y & z \end{vmatrix} = (2xz + 3y^2)\bar{i} - (z - 3xy)\bar{j} + (-y - 2x^2)\bar{k}$$

$$= \underset{(M)}{(2xz + 3y^2)}\bar{i} + \underset{(N)}{(3xy - z)}\bar{j} + \underset{(P)}{(-2x^2 - y)}\bar{k}$$

$$\text{Rotor}(\bar{F}\times\bar{G}) = (-1+1)\bar{i} - (-4x - 2x)\bar{j} + (3y - 6y)\bar{k}$$

$$\text{Rotor}(\bar{F}\times\bar{G}) = 0\bar{i} + 6x\bar{j} - 3y\bar{k}$$

b).- $\text{Rotor}(\bar{F}) = (3-0)\bar{i} - (0-0)\bar{j} + (2-0)\bar{k}$

$$\text{Rotor}(\bar{F}) = 3\bar{i} + 0\bar{j} + 2\bar{k}$$

c).- $\text{Rotor}(\bar{G}) = (0-0)\bar{i} - (0-0)\bar{j} + (0-0)\bar{k}$

$$\text{Rotor}(\bar{G}) = 0\bar{i} + 0\bar{j} + 0\bar{k}$$

$$\text{div}(\text{Rotor}(G)) = 0$$

d).- $\operatorname{div}(\bar{F}\times\bar{G}) = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} = 2z + 3x + 0$

$\operatorname{div}(\bar{F}\times\bar{G}) = 3x + 2z$

• Inciso 2 → Idem 1 .- REALIZARLO COMO TAREA!

PROBLEMA 6:

$\bar{F}(x,y,z) = \underset{(M)}{f(x)}\,\bar{i} + \underset{(N)}{g(y)}\,\bar{j} + \underset{(P)}{h(z)}\,\bar{k}$ Es IRROTACIONAL => ROTOR = 0

$\operatorname{Rotor}(\bar{F}) = (0-0)\,\bar{i} - (0-0)\,\bar{j} + (0-0)\,\bar{k} = 0$ => IRROTACIONAL!.-

PROBLEMA 7:

$\bar{F}(x,y,z) = \underset{(M)}{f(y,z)}\,\bar{i} + \underset{(N)}{g(x,z)}\,\bar{j} + \underset{(P)}{h(x,y)}\,\bar{k}$; Es INCOMPRESIBLE => DIVERGENCIA = 0!..

$\operatorname{div}(\bar{F}) = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} = 0+0+0 = 0$ => INCOMPRESIBLE!..